Transient calculation of electromagnetic field for grounding system based on consideration of displacement current

M. Trlep, M. Jesenik, A. Hamler, Members IEEE
University of Maribor, Faculty of Electrical Engineering and Computer Science
Smetanova ul. 17, 2000 Maribor, Slovenia
E-mail: mladen.trlep@uni-mb.si

Abstract — This paper shows the use of FEM for a second order time dependent electromagnetic field problem, around grounding systems (GS). Twenty-node isoparametric quadratic 3D finite element, three-node quadratic 1D finite element and a spatial transformation of the "infinite space" into the finite space are all applied to achieve better accuracy. Time integration is conducted with the Newmark algorithm. The applied program solution is suitable for any GS and isotropic/anisotropic soil properties as well as time-varying fault current.

I. INTRODUCTION

The primary goal of GS is to ensure the safety of personnel and prevent damage of installations. Their secondary goal is to provide a common reference voltage for all interconnected electrical and electronic systems. The program tool that is able to simulate the transient performance of grounding systems is fundamental, because it enables the optimization of the GS design, as well as the minimization of the disturbance level in the protected area. For that very reason, the goal of this research is to develop the methodology which allows a complete three-dimensional transient calculation of electromagnetic field, including the displacement current.

So far, three basic concepts have been used to simulate the transient performance of grounding arrangements: the circuit approach, the transmission line approach, and the electromagnetic field approach [1]. In this paper, the solution to analyze the transient behavior of grounding system is based on the electromagnetic field theory and on the implementation of FEM. The validity of the suggested method of analysis has been verified by the comparison of obtained results with the numerical and experimental results found in [7].

II. FEM MODEL OF TRANSIENT ELECTROMAGNETIC FIELD WITH CONSIDERATION OF DISPLACEMENT CURRENT

The governing partial differential equation for transient problems of GS can be derived from Maxwell's equations. When magnetic vector potential $\boldsymbol{A}$, electric scalar potential φ and displacement current are introduced to the conductive domain the following equation is obtained:

$$\nabla\times\frac{1}{\mu}\nabla\times\boldsymbol{A}-\nabla\frac{1}{\mu}\nabla\boldsymbol{A}+\sigma\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi\right)+\varepsilon\left(\frac{\partial^2\boldsymbol{A}}{\partial t^2}+\nabla\frac{\partial\varphi}{\partial t}\right)=0 \quad (1a)$$

$$\nabla\cdot\left[\sigma\left(\frac{\partial\boldsymbol{A}}{\partial t}+\nabla\varphi\right)+\varepsilon\left(\frac{\partial^2\boldsymbol{A}}{\partial t^2}+\nabla\frac{\partial\varphi}{\partial t}\right)\right]=0 \quad (1b)$$

The following equation can be written for the non-conductive domain:

$$\nabla\times\frac{1}{\mu}\nabla\times\boldsymbol{A}-\nabla\frac{1}{\mu}\nabla\boldsymbol{A}=0\,. \quad (2)$$

Where μ is the permeability, σ the electrical conductivity and ε is the permittivity. Equations (1.a) and (1.b) already contain the Coulumb's gauge to ensure the unique solution to the magnetic vector potential $\boldsymbol{A}$, which is given in greater detail in [2]. Equation (1.b) defines a well-known relation: divergence of the total current that occurs in the conductive domain is equal to zero. By applying the finite elements procedure and weighted residual method [2], the following equations for conductive domain are obtained (3a, 3b). For the non-conductive domain the equation (4) is given:

$$\int_{\Omega}\left[\frac{1}{\mu}\nabla\times\boldsymbol{N}_{\mathrm{i}}\,\nabla\times\boldsymbol{A}+\frac{1}{\mu}\nabla\boldsymbol{N}_{\mathrm{i}}\,\nabla\boldsymbol{A}+\sigma\,\boldsymbol{N}_{\mathrm{i}}\frac{\partial\boldsymbol{A}}{\partial t}+\sigma\,\boldsymbol{N}_{\mathrm{i}}\,\nabla\varphi+\varepsilon\,\boldsymbol{N}_{\mathrm{i}}\frac{\partial^2\boldsymbol{A}}{\partial t^2}+\varepsilon\,\boldsymbol{N}_{\mathrm{i}}\,\nabla\frac{\partial\varphi}{\partial t}\right]\mathrm{d}\Omega=0 \quad (3a)$$

$$\int_{\Omega}\left[\sigma\,\boldsymbol{N}_{\mathrm{i}}\frac{\partial\boldsymbol{A}}{\partial t}+\sigma\,\boldsymbol{N}_{\mathrm{i}}\,\nabla\varphi+\varepsilon\,\boldsymbol{N}_{\mathrm{i}}\frac{\partial^2\boldsymbol{A}}{\partial t^2}+\varepsilon\,\boldsymbol{N}_{\mathrm{i}}\,\nabla\frac{\partial\varphi}{\partial t}\right]\mathrm{d}\Omega=0 \quad (3b)$$

$$\int_{\Omega}\left[\frac{1}{\mu}\nabla\times\boldsymbol{N}_{\mathrm{i}}\,\nabla\times\boldsymbol{A}+\frac{1}{\mu}\nabla\boldsymbol{N}_{\mathrm{i}}\,\nabla\boldsymbol{A}\right]\mathrm{d}\Omega=0\cdot \quad (4)$$

The described problem is an open boundary problem. Therefore, the numerical model includes the spatial transformation [3], which divides the total domain of the open boundary problem onto a non-transformed inner domain and the transformed outer (infinite) domain. In order to get an accurate field calculation, the soil and the air in the transformed and non-transformed domain of the problem are discretized by 20 nodes second order 3D finite elements. The conductors of the grounding grid or rods (in the non-transformed domain) are discretized by the 3 nodes second order 1D finite elements [6], [8]. The final FEM equation is represented by a system of second order ordinary differential equations (5), where V (6) represents the modified electric scalar potential [2], in order to ensure that matrices $\boldsymbol{C}$ and $\boldsymbol{M}$ are symmetrical.

$$[K]\begin{Bmatrix}\boldsymbol{A}\\ V\end{Bmatrix}+\sigma[C]\begin{Bmatrix}\dot{\boldsymbol{A}}\\ \dot{V}\end{Bmatrix}+\varepsilon[M]\begin{Bmatrix}\ddot{\boldsymbol{A}}\\ \ddot{V}\end{Bmatrix}=0 \quad (5)$$

$$\varphi = \frac{\partial V}{\partial t} = \dot{V} \; . \tag{6}$$

The column vector of unknown nodal potentials in (5) is $\{\boldsymbol{A},\ V\}$. Next column vectors are the first and the second derivative of nodal potentials and $\boldsymbol{K}$, $\boldsymbol{C}$, $\boldsymbol{M}$ are corresponding matrices, which are linked with the potentials $\boldsymbol{A}$ and V, and the Laplacian operator ($\boldsymbol{K}$); with the induced conducting current ($\boldsymbol{C}$) and with the displacement current ($\boldsymbol{M}$). Time integration (5) can be conducted with different time-step algorithms such as Newmark's, Crank-Nicolson's, Wilson's and others [8]. With an assumption of linear interpolation throughout time [8], and with the use of the Newmark algorithm, the following recursive equation (7) is obtained from (5):

$$\left[\left(f_M + \frac{1}{\theta \Delta t}\right)[M] + \theta \Delta t [K]\ \right]\begin{Bmatrix} \boldsymbol{A}^{(n+1)} \\ V^{(n+1)} \end{Bmatrix} = \left[\left(f_M + \frac{1}{\theta \Delta t}\right)[M]\right]\begin{Bmatrix} \boldsymbol{A}^{(n)} \\ V^{(n)} \end{Bmatrix} - \\ -\left[(1-\theta)\Delta t [K]\ \right]\begin{Bmatrix} \boldsymbol{A}^{(n)} \\ V^{(n)} \end{Bmatrix} + \left[\frac{1}{\theta}[M]\right]\frac{d}{dt}\begin{Bmatrix} \boldsymbol{A}^{(n)} \\ V^{(n)} \end{Bmatrix} \tag{7}$$

where parameters f_M and θ are given by the following expressions: $f_M=\mu/\varepsilon$ and θ=0,5. The recursive equation (7) enables the calculation of the potential, in the new time step (n+1), depending on the preceding time step (n).

III. Application and Results

The program solution has been tested on differently GS, in order to verify the reliability, which will be presented in the full paper. The application of the program solution for the case from [7] is shown in the following text. The description contains a detailed and extensive analysis of the conducted grounding grid impulse characteristics. The considered grounding grid is of a rectangular type with dimensions 10 m × 10 m. It is made out of round copper conductors that are 50 mm^2 in cross section. There are four meshes in a grid, each measuring 5 m × 5 m, as shown in Fig. 1. The grounding grid is buried 0,5 m horizontally below the earth surface into a two-layer soil with the upper-layer resistivity (from earth surface to the depth of 0,6 m) of 50 Ω·m and the bottom-layer resistivity of 20 Ω·m.

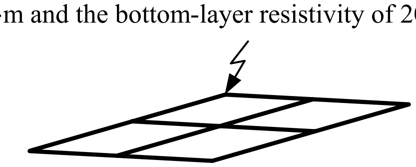

Fig.1. Schematic view of the grounding grid under analysis.

The application of our FEM solution is presented to analyze the behaviour of the grounding grid when fed by an injected time variable potential function with maximum voltage of 14,8 V at the discharge point, and the ratio between the nominal time increase and time to half-value the impulse voltage front T1/T2 = 8μs/77μs respectively. In Fig. 2, the comparison of calculated current variation, obtained as a response to the shape of the injected potential function, with current variation from [7] is presented.

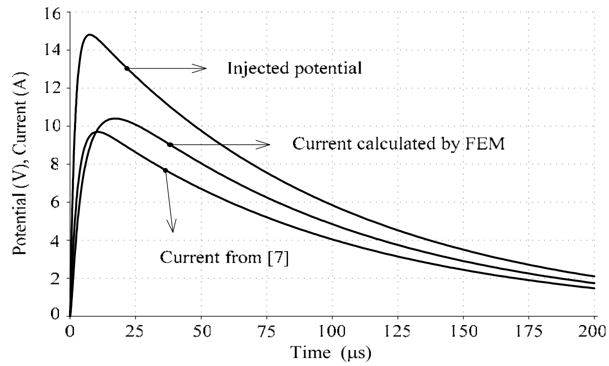

Fig. 2. The transient voltage and currents of the grounding grid.

Fig.3 shows the potential distribution on the earth surface above the grounding grid. Note that the result is shown in a particular moment of time, 2 μs, after the fault current starts to flow throughout the grounding grid.

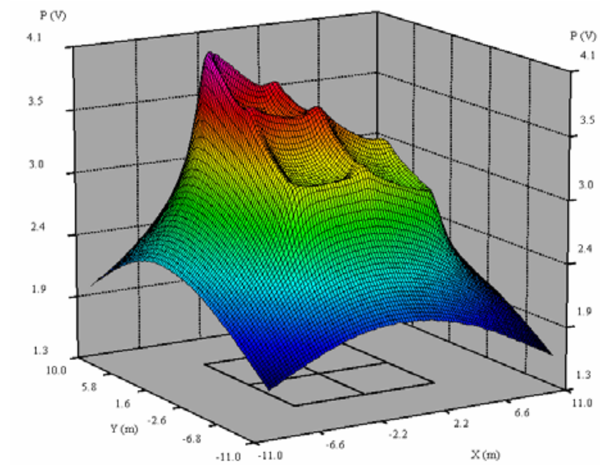

Fig. 3. Distribution of the electric potentials on earth surface.

IV. References

[1] L. Grcev and F. Dawalibi, "An electromagnetic model for transients in grounding systems," IEEE Transactions on Power Delivery, vol. 5, no. 4, pp. 1773-1781, October 1990.

[2] O. Biro and K. Preis, "On the use of the magnetic vector potential in the finite element analysis of the three dimensional eddy current," IEEE Transactions on Magnetics, vol. 25, no. 4, pp. 3145-3159, July 1989.

[3] A. Stochniol, "A general transformation for open boundary finite element method for electromagnetic problems", IEEE Transsaction on Magnetics, vol. 28, no. 2, pp. 1679-1681, March 1992.

[4] B. Nekhoul, C. Guerin, P. Labie, G. Meunier, and R. Feuillet, "A Finite element method for calculating the electromagnetic fields generated by substation grounding systems," IEEE Transactions on Magnetics, vol. 31, no. 3, pp. 2150-2153, May 1995.

[5] B. Nekhoul, P. Labie, F. X. Zgainski, and G. Meunier, "Calculating the impedance of a grounding system," IEEE Transactions on Magnetics, vol. 32, no. 3, pp. 1509-1512, May 1996.

[6] M. Trlep, A. Hamler, and B. Hribernik, "The analysis of complex grounding systems by FEM," IEEE Transactions on Magnetics, vol. 34, pp. 2521-2524, September 1998.

[7] Z. Stojković, M.S. Savić, J.M. Nahman, D. Salamon, B. Bukorović, "Sensitivity Analysis of Experimentally Determined Grounding Grid Impulse Characteristics," IEEE Transactions on Power Delivery, vol. 13, no. 4, pp. 1136-1141, October 1998.

[8] I.M. Smith, D. V. Griffiths, " Programming the Finite Element Methods," 4th Edition, John Wiley & Sons, Ltd., 2004.